

STRAIGHT LINES [JEE ADVANCED PREVIOUS YEAR SOLVED PAPER]

JEE ADVANCED

Single Correct Answers Type

1. The straight lines $x + y = 0$, $3x + y - 4 = 0$, and $x + 3y - 4 = 0$ form a triangle which is

- a. isosceles
- c. right-angled

- b. equilateral
- d. none of these

(IIT-JEE 1983)

2. If $P \equiv (1, 0)$, $Q \equiv (-1, 0)$, and $R \equiv (2, 0)$ are three given points, then the locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is
 a. a straight line parallel to the x -axis
 b. a circle passing through the origin
 c. a circle with center at the origin
 d. a straight line parallel to the y -axis (IIT-JEE 1988)
3. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q . Then,
 a. $a^2 + b^2 = p^2 + q^2$ b. $1/a^2 + 1/b^2 = 1/p^2 + 1/q^2$
 c. $a^2 + p^2 = b^2 + q^2$ d. $1/a^2 + 1/p^2 = 1/b^2 + 1/q^2$
 (IIT-JEE 1990)
4. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is
 a. square b. circle
 c. straight line d. two intersecting lines
 (IIT-JEE 1992)
5. The equations to a pair of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. The equations to its diagonals are
 a. $x + 4y = 13, y = 4x - 7$
 b. $4x + y = 13, 4y = x - 7$
 c. $4x + y = 13, y = 4x - 7$
 d. $y - 4x = 13, y + 4x = 7$
 (IIT-JEE 1994)
6. The orthocenter of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is
 a. $(1/2, 1/2)$ b. $(1/3, 1/3)$
 c. $(0, 0)$ d. $(1/4, 1/4)$
 (IIT-JEE 1995)
7. Let PQR be a right-angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is
 a. $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
 b. $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
 c. $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
 d. $3x^2 - 3y^2 - 8xy - 15y - 20 = 0$ (IIT-JEE 1999)
8. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$, and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is
 a. $2x - 9y - 7 = 0$ b. $2x - 9y - 11 = 0$
 c. $2x + 9y - 11 = 0$ d. $2x + 9y + 7 = 0$
 (IIT-JEE 2000)
9. The number of integral values of m , for which the x -coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is
 a. 2 b. 0 c. 4 d. 1
 (IIT-JEE 2001)
10. The area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$, and $y = nx + 1$ equals
 a. $|m + n|/(m - n)^2$ b. $2/|m + n|$
 c. $1/(|m + n|)$ d. $1/(|m - n|)$
 (IIT-JEE 2001)
11. Let $P = (-1, 0)$, $Q = (0, 0)$, and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of $\angle PQR$ is
 a. $(\sqrt{3}/2)x + y = 0$ b. $x + \sqrt{3}y = 0$
 c. $\sqrt{3}x + y = 0$ d. $x + (\sqrt{3}/2)y = 0$
 (IIT-JEE 2002)
12. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q , respectively. Then the point O divides the segment PQ in the ratio
 a. 1 : 2 b. 3 : 4
 c. 2 : 1 d. 4 : 3 (IIT-JEE 2002)
13. Area of the triangle formed by the line $x + y = 3$ and the angle bisectors of the pairs of straight lines $x^2 - y^2 + 2y = 1$ is
 a. 2 sq. units b. 4 sq. units
 c. 6 sq. units d. 8 sq. units
 (IIT-JEE 2004)
14. The locus of the orthocenter of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0$, $(1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$, is
 a. a hyperbola b. a parabola
 c. an ellipse d. a straight line
 (IIT-JEE 2009)
15. A straight line L through the point $(3, -2)$ is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x -axis, then the equation of L is
 a. $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$
 b. $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
 c. $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$
 d. $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$
 (IIT-JEE 2011)
16. For $a > b > c > 0$, the distance between $(1, 1)$ and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then
 a. $a + b - c > 0$ b. $a - b + c < 0$
 c. $a - b + c > 0$ d. $a + b - c < 0$
 (JEE Advanced 2013)

Multiple Correct Answers Type

1. Three lines $px + qy + r = 0$, $qx + ry + p = 0$, and $rx + py + q = 0$ are concurrent if
 a. $p + q + r = 0$ b. $p^2 + q^2 + r^2 = pr + rp + pq$
 c. $p^3 + q^3 + r^3 = 3pqr$ d. none of these
 (IIT-JEE 1985)
2. All points lying inside the triangle formed by the points $(1, 3)$, $(5, 0)$, and $(-1, 2)$ satisfy
 a. $3x + 2y \geq 0$ b. $2x + y - 13 \geq 0$
 c. $2x - 3y \leq 12$ d. $-2x + y \geq 0$
 (IIT-JEE 1986)
3. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a
 a. rectangle b. square
 c. cyclic quadrilateral d. rhombus
 (IIT-JEE 1998)



Matching Column Type

1. Consider the lines given by

$$L_1: x + 3y - 5 = 0$$

$$L_2: 3x - ky - 1 = 0$$

$$L_3: 5x + 2y - 12 = 0$$

Column I	Column II
(a) L_1, L_2, L_3 are concurrent if	(p) $k = -9$
(b) One of L_1, L_2, L_3 is parallel to at least one of the other two if	(q) $k = -6/5$
(c) L_1, L_2, L_3 form a triangle if	(r) $k = 5/6$
(d) L_1, L_2, L_3 do not form a triangle if	(s) $k = 5$

(IIT-JEE 2008)

Assertion-Reasoning Type

1. Lines L_1, L_2 given by $y - x = 0$ and $2x + y = 0$ intersect the line L_3 given by $y + 2 = 0$ at P and Q , respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R .

Statement 1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

Statement 2: In any triangle, the bisector of an angle divides the triangle into two similar triangles.

- Both the statements are true and statement 2 is the correct explanation of statement 1.
- Both the statements are true but statement 2 is not the correct explanation of statement 1.
- Statement 1 is true and statement 2 is false.
- Statement 1 is false and statement 2 is true.

(IIT-JEE 2007)

Fill in the Blanks Type

- The area enclosed within the curve $|x| + |y| = 1$ is _____. (IIT-JEE 1981)
- The set of lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$, is concurrent at the point _____. (IIT-JEE 1982)
- If a, b , and c are in AP, then the straight line $ax + by + c = 0$ will always pass through a fixed point whose coordinates are _____. (IIT-JEE 1984)
- The orthocenter of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$, and $4x - y + 4 = 0$ lies in quadrant number _____. (IIT-JEE 1985)
- Let the algebraic sum of the perpendicular distance from the points $(2, 0)$, $(0, 2)$, and $(1, 1)$ to a variable straight line be zero. Then the line passes through a fixed point whose coordinates are _____. (IIT-JEE 1991)
- The vertices of a triangle are $A(-1, -7)$, $B(5, 1)$, and $C(1, 4)$. The equation of the bisector of $\angle ABC$ is _____. (IIT-JEE 1993)

True/False Type

- The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$. (IIT-JEE 1983)
- The lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the coordinate axes at concyclic points. (IIT-JEE 1988)

Subjective Type

- One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are $(-3, 1)$ and $(1, 1)$. Find the equations of the other three sides. (IIT-JEE 1978)
- Find the equation of the line which bisects the obtuse angle between the lines $x - 2y + 4 = 0$ and $4x - 3y + 2 = 0$. (IIT-JEE 1979)
- A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by line L and the coordinate axes is 5. Find the equation of line L . (IIT-JEE 1980)
- The points $(1, 3)$ and $(5, 1)$ are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. Find c and the remaining vertices. (IIT-JEE 1981)
- The ends A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY , respectively. If the rectangle $OAPB$ is completed, then show that the locus of the foot of the perpendicular drawn from P to AB is $x^{2/3} + y^{2/3} = c^{2/3}$. (IIT-JEE 1983)
- Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side. (IIT-JEE 1984)
- One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If A and B are the points $(-3, 4)$ and $(5, 4)$, respectively, then find the area of the rectangle. (IIT-JEE 1985)
- Two sides of a rhombus $ABCD$ are parallel to the lines $y = x + 2$ and $y = 7x + 3$. If the diagonals of the rhombus intersect at the point $(1, 2)$ and the vertex A is on the y -axis, then find the possible coordinates of A . (IIT-JEE 1985)
- The equations of the perpendicular bisectors of the sides AB and AC of triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$, respectively. If the point A is $(1, -2)$, then find the equation of the line BC . (IIT-JEE 1986)
- Lines $L_1 \equiv ax + by + c = 0$ and $L_2 \equiv lx + my + n = 0$ intersect at the point P and make an angle θ with each other. Find the equation of a line different from L_2 which passes through P and makes the same angle θ with L_1 . (IIT-JEE 1988)
- Let ABC be a triangle with $AB = AC$. If D is the midpoint of BC , E is the foot of the perpendicular drawn from D to AC , and F is the midpoint of DE , then prove that AF is perpendicular to BE . (IIT-JEE 1989)

12. Straight lines $3x + 4y = 5$ and $4x - 3y = 15$ intersect at the point A . Points B and C are chosen on these two lines such that $AB = AC$. Determine the possible equations of the line BC passing through the point $(1, 2)$.
(IIT-JEE 1990)
13. Find the equation of the line passing through the point $(2, 3)$ and making an intercept of length 2 units between the lines $y + 2x = 3$ and $y + 2x = 5$.
(IIT-JEE 1991)
14. Show that all chords of the curve $3x^2 - y^2 - 2x + 4y = 0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point.
(IIT-JEE 1991)
15. Determine all the values of α for which the point (α, α^2) lies inside the triangle formed by the lines
 $2x + 3y - 1 = 0$
 $x + 2y - 3 = 0$
 $5x - 6y - 1 = 0$
(IIT-JEE 1992)
16. A line through $A(-5, -4)$ meets the lines $x + 3y + 2 = 0$, $2x + y + 4 = 0$, and $x - y - 5 = 0$ at the points B, C , and D , respectively. If $(15/AB)^2 + (10/AC)^2 = (6/AD)^2$, find the equation of the line.
(IIT-JEE 1993)
17. A rectangle $PQRS$ has its side PQ parallel to the line $y = mx$ and vertices P, Q , and S on the lines $y = a$, $x = b$, and $x = -b$, respectively. Find the locus of the vertex R .
(IIT-JEE 1996)
18. A straight line L through the origin meets the lines $x + y = 1$ and $x + y = 3$ at P and Q , respectively. Through P and Q , two straight lines L_1 and L_2 are drawn, parallel to $2x - y = 5$ and $3x + y = 5$, respectively. Lines L_1 and L_2 intersect at R . Show that the locus of R , as L varies, is a straight line.
(IIT-JEE 2002)
19. A straight line L with negative slope passes through the point $(8, 2)$ and cuts the positive coordinate axes at points P and Q . Find the absolute minimum value of $OP + OQ$ as L varies, where O is the origin.
(IIT-JEE 2002)
20. The area of the triangle formed by the intersection of a line parallel to the x -axis and passing through $P(h, k)$ with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P .
(IIT-JEE 2005)

Answer Key

JEE Advanced

Single Correct Answer Type

1. a. 2. d. 3. b. 4. a. 5. c.
6. c. 7. b. 8. d. 9. a. 10. d.
11. c. 12. b. 13. a. 14. d. 15. b.
16. a.

Multiple Correct Answers Type

1. a., b., c. 2. a., c. 3. d.

Matching Column Type

1. (a)-(s); (b)-(p), (q); (c)-(r); (d)-(p), (q), (s)

Assertion-Reasoning Type

1. c.

Fill in the Blanks Type

1. 2 sq. units 2. $(3/4, 1/2)$
3. $(1, -2)$ 4. first quadrant
5. $(1, 1)$ 6. $x - 7y + 2 = 0$

True/False Type

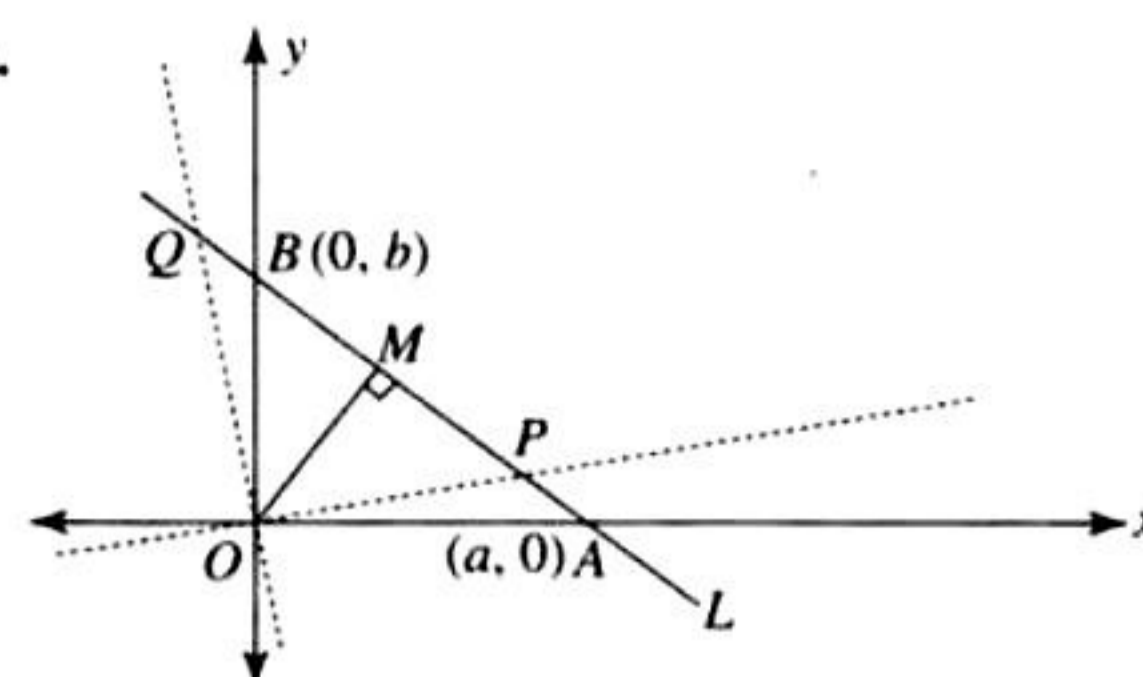
1. True 2. True

Subjective Type

1. $4x + 7y - 11 = 0$, $7x - 4y - 3 = 0$, $7x - 4y + 25 = 0$
2. $(4 - \sqrt{5})x + (2\sqrt{5} - 3)y - (4\sqrt{5} - 2) = 0$
3. $x + 5y - 5\sqrt{2} = 0$ or $x + 5y + 5\sqrt{2} = 0$
4. $c = -4, (4, 4), (2, 0)$
5. $x - 3y - 31 = 0, 3x + y + 7 = 0$
6. 32 sq. units 8. $(0, 0)$ or $(0, 5/2)$
7. $14x + 23y - 40 = 0$
9. $2(al + bm)(ax + by + c) - (a^2 + b^2)(lx + my + n) = 0$
10. $x - 7y + 13 = 0$ or $7x + y - 9 = 0$
11. $3x + 4y - 18 = 0, x = 2$
12. $x \in (-3/2, -1) \cup (1/2, 1)$
13. $2x + 3y + 22 = 0$
14. $(m^2 - 1)x - my + b(m^2 + 1) + am = 0$
15. 18
16. $y = 2x + 1$ or $y = -2x + 1$

Hints and Solutions

3. b.



As L has intercepts a and b on the axes, the equation of L is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad (i)$$

Let the x -axis and the y -axis be rotated through an angle θ in the anticlockwise direction. In the new system, the intercepts are p and q ($OP = p$, $OQ = q$). Therefore, the equation of L w.r.t. new coordinate system becomes

$$\frac{x}{p} + \frac{y}{q} = 1 \quad (ii)$$

As the origin is fixed in rotation, the distance of line from the origin in both the cases should be same. Hence, we get

$$d = \left| \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| = \left| \frac{1}{\sqrt{\frac{1}{p^2} + \frac{1}{q^2}}} \right|$$

$$\text{or } \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$$

4. a. Let the two perpendicular lines be the coordinate axes. Let (x, y) be the point, sum of whose distances from two axes is 1.

Then we must have

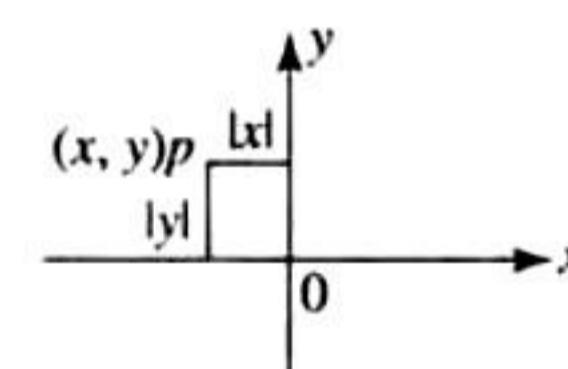
$$|x| + |y| = 1$$

$$\text{or } \pm x \pm y = 1$$

The four lines are

$$x + y = 1, x - y = 1, -x + y = 1, -x - y = 1$$

Any two adjacent sides are perpendicular to each other. Also, each line is equidistant from the origin. Therefore, the figure formed is a square.



5. c. The sides of parallelogram are $x = 2$, $x = 3$, $y = 1$, $y = 5$.

\therefore Vertices of parallelograms are as shown in the figure

The equation of diagonal AC is

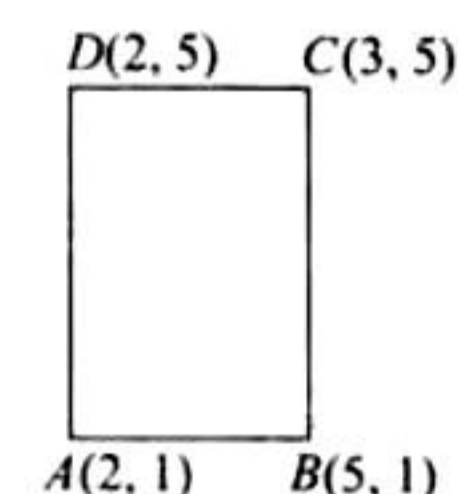
$$\frac{y-1}{5-1} = \frac{x-2}{3-2}$$

$$\text{or } y = 4x - 7$$

and the equation of diagonal BD is

$$\frac{x-2}{3-2} = \frac{y-5}{1-5}$$

$$\text{or } 4x + y = 13$$



6. c. The lines by which triangle is formed are $x = 0$, $y = 0$, and $x + y = 1$. Clearly, it is a right triangle. We know that in a right-angled triangle, the orthocenter coincides with the vertex at which right angle is formed. Therefore, the orthocenter is $(0, 0)$.

JEE Advanced

Single Correct Answer Type

1. a. Solving the given equations of lines pairwise, we get the vertices of triangle as $A(-2, 2)$, $B(2, -2)$, $C(1, 1)$. Then,

$$AB = \sqrt{16 + 16} = 4\sqrt{2}$$

$$BC = \sqrt{1 + 9} = \sqrt{10}$$

$$CA = \sqrt{9 + 1} = \sqrt{10}$$

Hence, the triangle is isosceles.

2. d. We have $P \equiv (1, 0)$, $Q \equiv (-1, 0)$, $R \equiv (2, 0)$. Let $S \equiv (x, y)$.

Now, given that $SQ^2 + SR^2 = 2SP^2$. Hence,

$$(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$$

$$\text{or } 2x^2 + 2y^2 - 2x + 5 = 2x^2 + 2y^2 - 4x + 2$$

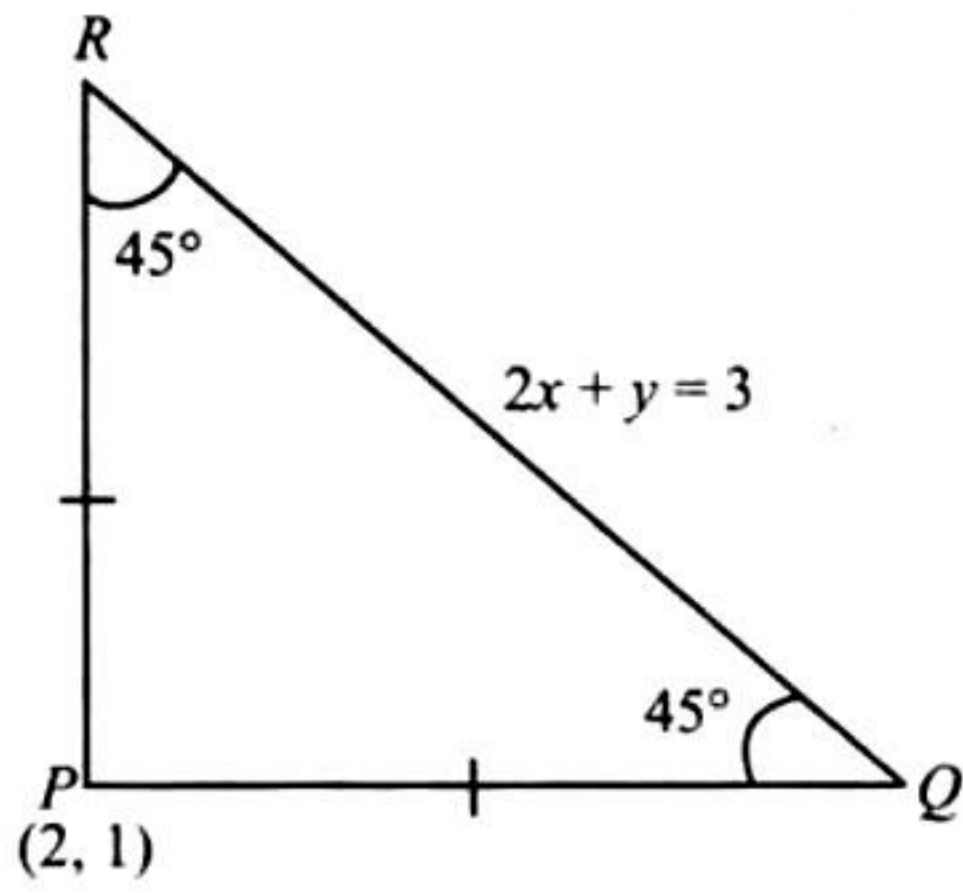
$$\text{or } 2x + 3 = 0$$

$$\text{or } x = -\frac{3}{2}$$

which is a straight line parallel to the y -axis.



7. b.



Let m be the slope of PQ . Then,

$$\tan 45^\circ = \left| \frac{m - (-2)}{1 + m(-2)} \right| = \left| \frac{m + 2}{1 - 2m} \right|$$

$$\therefore m + 2 = 1 - 2m \text{ or } -1 + 2m = m + 2$$

$$\therefore m = -\frac{1}{3} \text{ or } m = 3$$

Hence, the equation of PQ is

$$y - 1 = -\frac{1}{3}(x - 2)$$

$$\text{or } x + 3y - 5 = 0$$

and the equation of PR is

$$y - 1 = 3(x - 2)$$

$$\text{or } 3x - y - 5 = 0$$

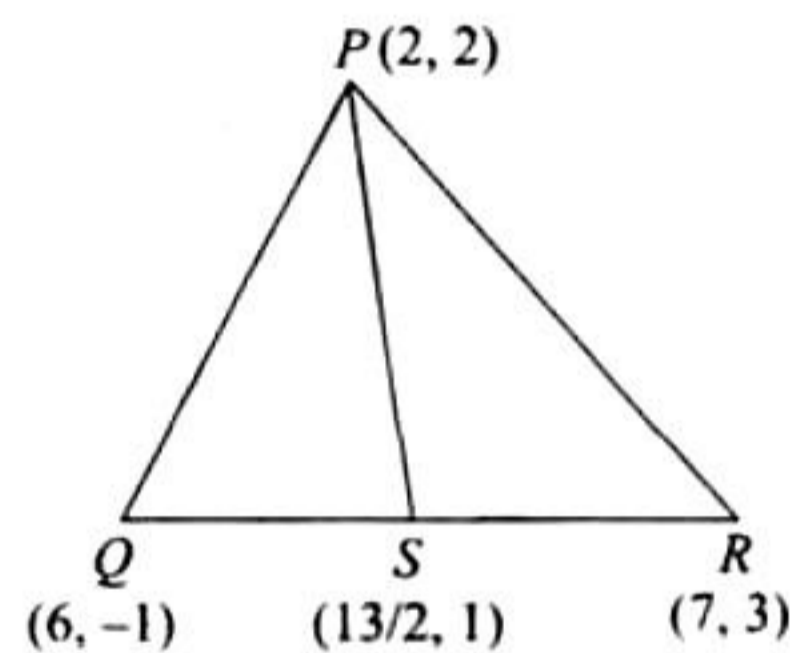
Hence, the combined equation of PQ and PR is

$$(x + 3y - 5)(3x - y - 5) = 0$$

$$\text{or } 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$$

8. d. S is the midpoint of Q and R . Therefore,

$$S \equiv \left(\frac{7+6}{2}, \frac{3-1}{2} \right) \equiv \left(\frac{13}{2}, 1 \right)$$



$$\text{Now, the slope of } PS \text{ is } m = \frac{2-1}{2-13/2} = -\frac{2}{9}$$

Then the equation of the line passing through $(1, -1)$ and parallel to PS is

$$y + 1 = -\frac{2}{9}(x - 1)$$

$$\text{or } 2x + 9y + 7 = 0$$

9. a. Solving given lines for the x -coordinate of the point of intersection

$$3x + 4(mx + 1) = 9$$

$$\text{or } (3 + 4m)x = 5 \text{ or } x = \frac{5}{3 + 4m}$$

For x to be an integer, $3 + 4m$ should be a divisor of 5, i.e., of 1, -1, 5, or -5. Hence,

$$3 + 4m = 1 \text{ or } m = -\frac{1}{2} \text{ (not integer)}$$

$$3 + 4m = -1 \text{ or } m = -1 \text{ (integer)}$$

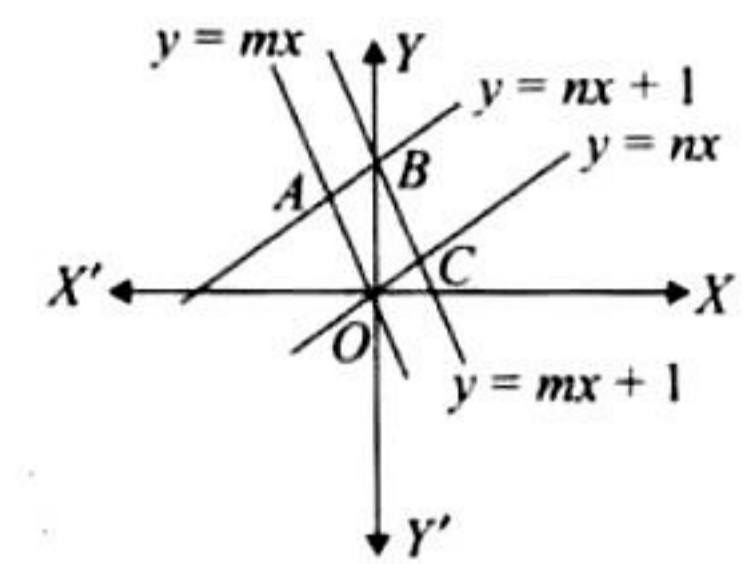
$$3 + 4m = 5 \text{ or } m = \frac{1}{2} \text{ (not an integer)}$$

$$3 + 4m = -5 \text{ or } m = -2 \text{ (integer)}$$

Hence, there are two integral values of m .

10. d. $y = mx$ and $y = nx$ are lines through $(0, 0)$, $y = mx + 1$ and $y = nx + 1$ are lines parallel to $y = mx$ and $y = nx$ respectively with y -intercept 1.

The vertices are $O(0, 0)$, $A(1/(m - n), m/(m - n))$. The area of parallelogram is given by



$$2 \times \text{Ar}(\Delta OAB)$$

$$= 2 \times \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1}{m-n} & \frac{m}{m-n} & 1 \end{vmatrix}$$

$$= \frac{1}{|m-n|}$$

11. c. The slope of QR is

$$(3\sqrt{3} - 0)/(3 - 0) = \sqrt{3}$$

$$\text{i.e., } \theta = 60^\circ$$

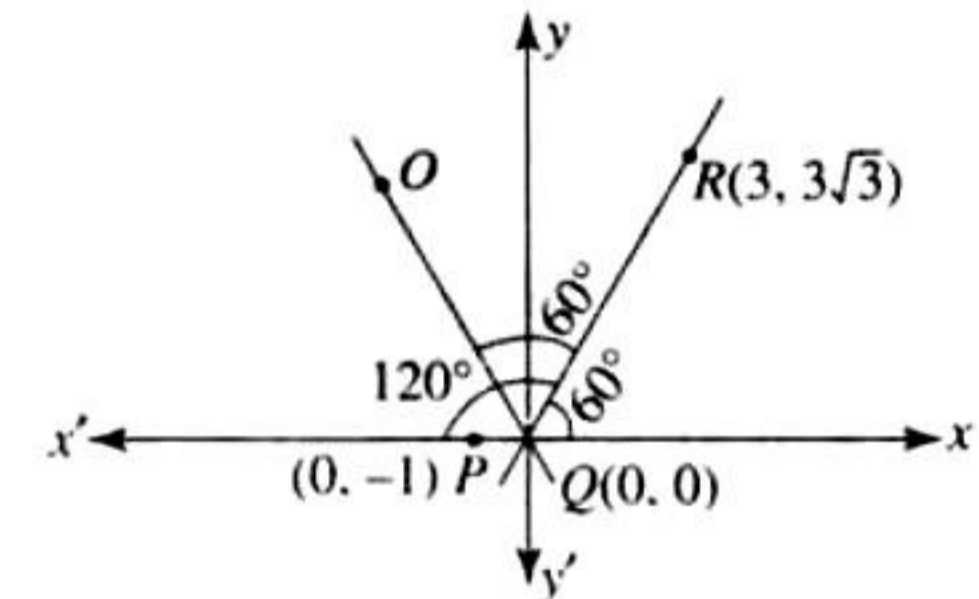
Clearly, $\angle PQR = 120^\circ$.

OQ is the angle bisector of the angle.

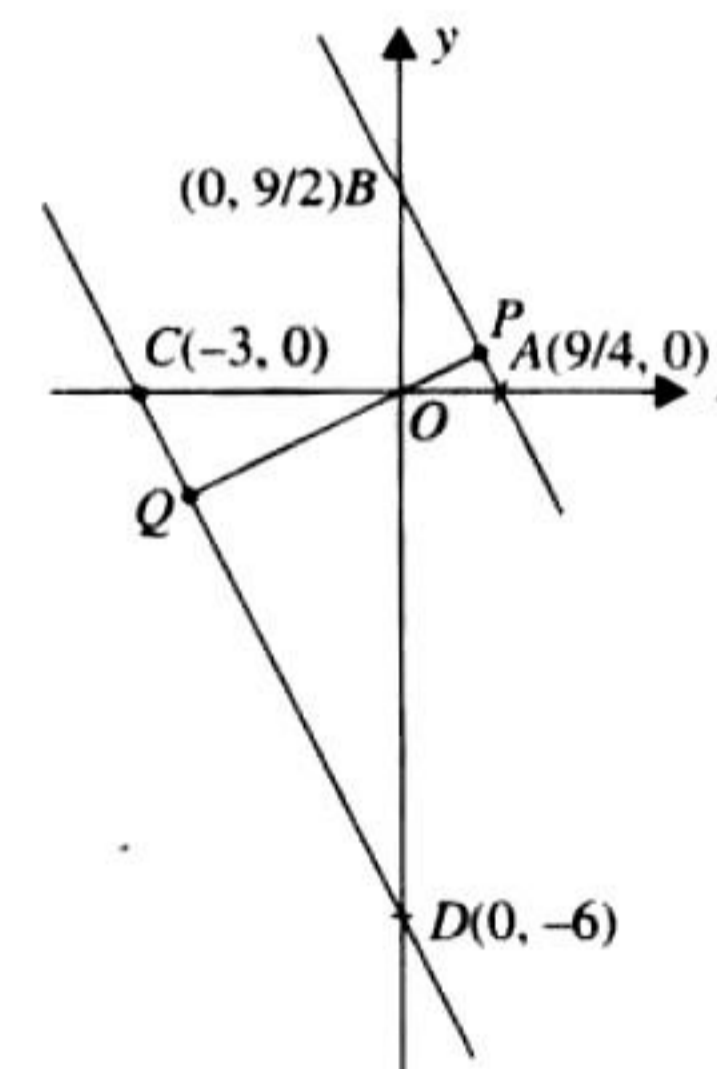
So, line OQ makes 120° with the positive direction of the x -axis.

Therefore, the equation of the bisector of $\angle PQR$ is

$$y = x \tan 120^\circ \text{ or } y = -\sqrt{3}x, \text{ i.e., } \sqrt{3}x + y = 0.$$



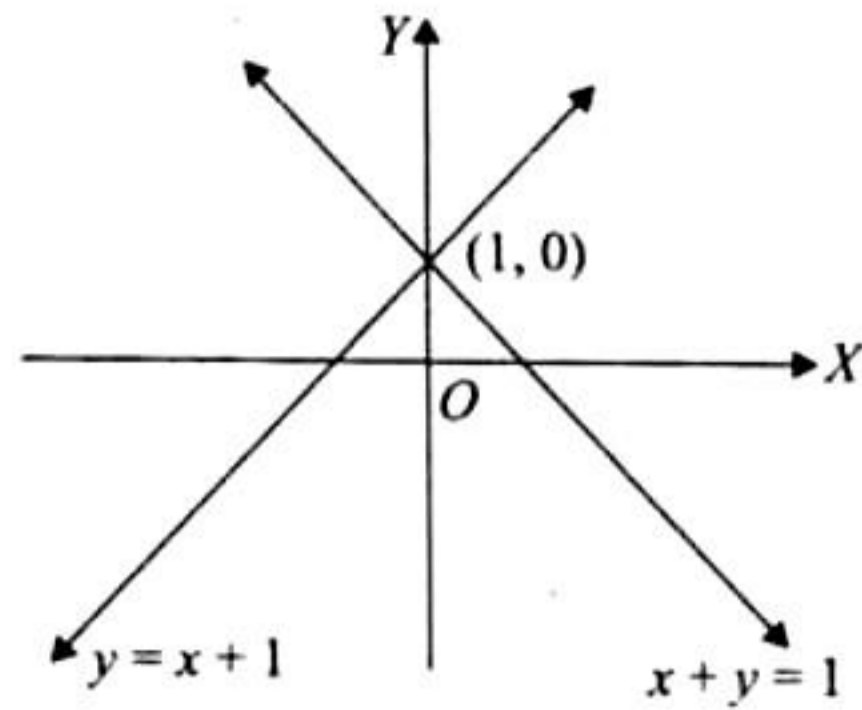
12. b. Let any line through the origin meets the given lines at P and Q as shown in figure.



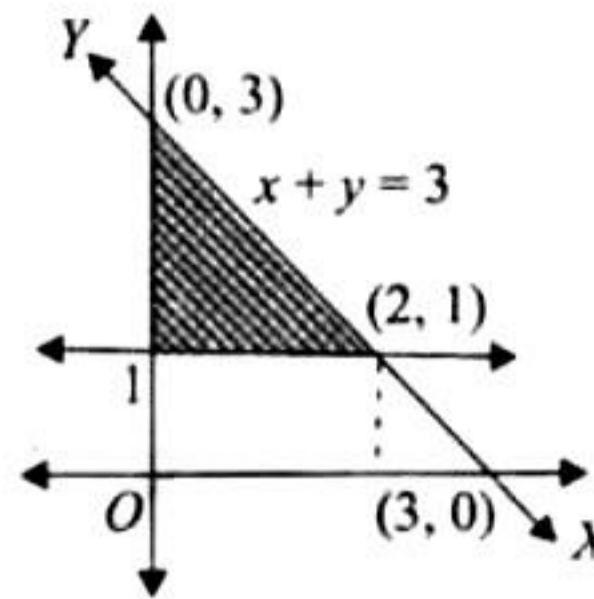
Now, from the figure, triangles OAP and OCQ are similar. Therefore,

$$\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$$

13. a. $x^2 - y^2 + 2y = 1$ or $x = \pm(y - 1)$



The bisectors of the above lines are $x = 0$ and $y = 1$.



So, the area between $x = 0$, $y = 1$, and $x + y = 3$ is the shaded region shown in the figure. The area is given by $(1/2) \times 2 \times 2 = 2$ sq. units.

14. d. The intersection point of $y = 0$ with the first line is $B(-p, 0)$.

The intersection point of $y = 0$ with the second line is $A(-q, 0)$.

The intersection point of the two lines is

$$C(pq, (p+1)(q+1))$$

The altitude from C to AB is $x = pq$.

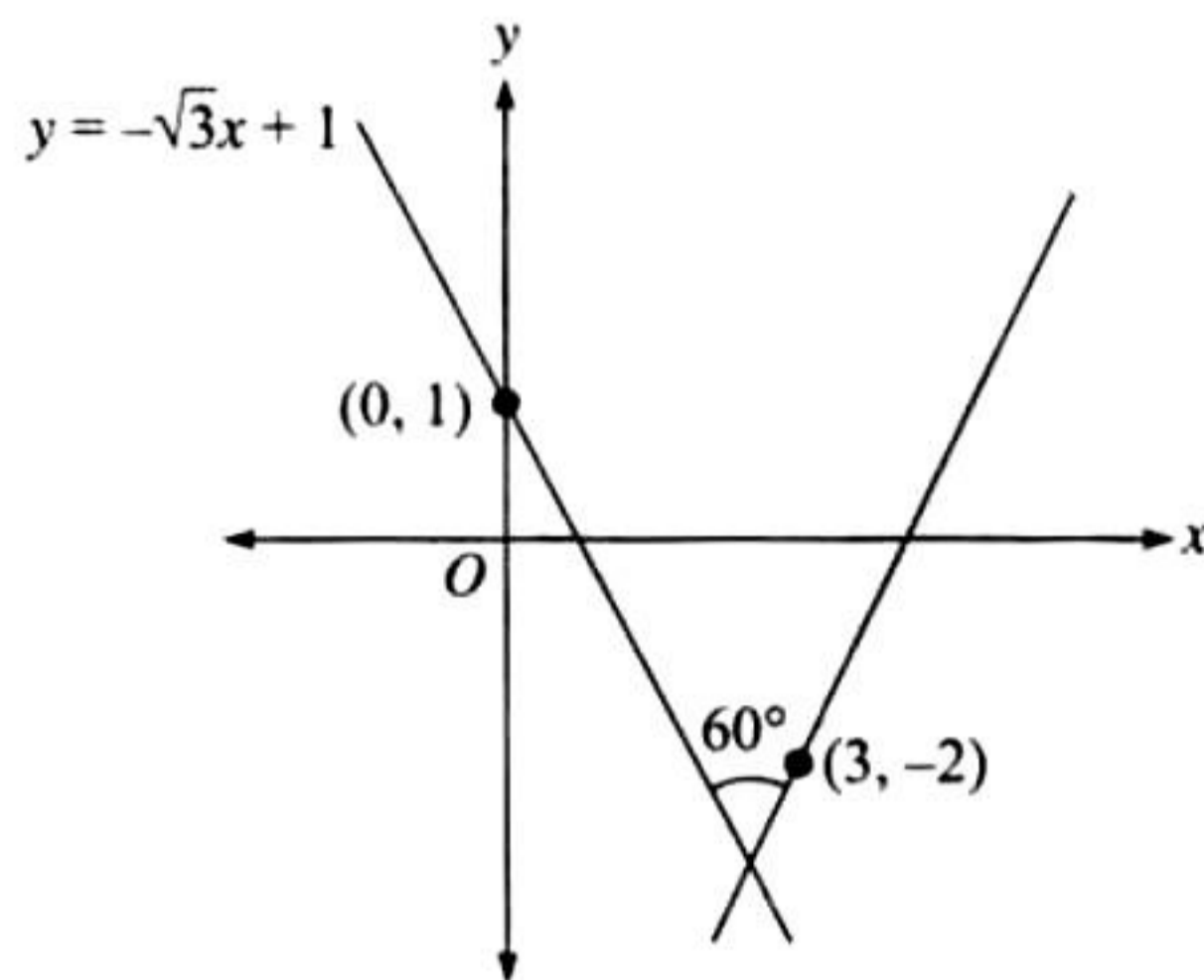
The altitude from C to AC is

$$y = -\frac{q}{1+q}(x+p)$$

Solving these two, we get $x = pq$ and $y = -pq$.

Therefore, the locus of the orthocenter is $x + y = 0$.

15. b.



Let the slope of the required line be m . Then

$$\left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right| = \sqrt{3}$$

$$\therefore m + \sqrt{3} = \pm(\sqrt{3} - 3m)$$

$$\therefore m = 0 \text{ or } m = \sqrt{3}$$

Therefore, the equation is

$$y + 2 = \sqrt{3}(x - 3) \text{ (} m \neq 0 \text{ as given that line cuts the } x\text{-axis)}$$

$$\text{or } \sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$$

16. a. Solving given lines for their point of intersection, we get the point of intersection as $(-c/(a+b), -c/(a+b))$.

Its distance from $(1, 1)$ is

$$\sqrt{\left(1 + \frac{c}{a+b}\right)^2 + \left(1 + \frac{c}{a+b}\right)^2} < 2\sqrt{2} \quad (\text{given})$$

$$\text{or } (a+b+c)^2 < 4(a+b)^2 \text{ or } (a+b+c)^2 - (2a+2b)^2 < 0$$

$$\text{or } (c-a-b)(c+3a+3b) < 0$$

Since $a > b > c > 0$, $(c-a-b) < 0$ or $a+b-c > 0$.

Multiple Correct Answers Type

1. a., b., c. For the concurrency of three lines $px + qy + r = 0$, $qx + ry + p = 0$, $rx + py + q = 0$, we must have

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$\text{or } 3pqr - p^3 - q^3 - r^3 = 0$$

$$\text{or } (p+q+r)(p^2+q^2+r^2-pq-pr-rq) = 0$$

2. a., c. Substituting the coordinates of the points $(1, 3)$, $(5, 0)$, and $(-1, 2)$ in $3x + 2y$, we obtain the values 9, 15, and 1, respectively which are all positive. Therefore, all the points lying inside the triangle formed by the given points satisfy $3x + 2y \geq 0$.

Substituting the coordinates of the given points in $2x + 3y - 13$, we find the values -2 , -3 , and -9 , respectively, which are all negative. So, (b) is not correct.

Again, substituting the given points in $2x - 3y - 12$, we get -19 , -2 , and -20 , respectively, which are all negative. It follows that all points lying inside the triangle formed by the given point satisfy $2x - 3y - 12 \leq 0$. So, (c) is the correct answer.

Finally, substituting the coordinates of the given point in $-2x + y$, we get 1, -10 , and 4, respectively, which are not all positive. So, (d) is not correct.

3. d. The slope of $x + 3y = 4$ is $-1/3$ and the slope of $6x - 2y = 7$ is 3. Therefore, these two lines are perpendicular, which shows that both diagonals are perpendicular. Hence, $PQRS$ must be a rhombus.

Matching Column Type

1. (a)-(s); (b)-(p), (q); (c)-(r); (d)-(p), (q), (s)

Given lines are

$$L_1 : x + 3y - 5 = 0$$

$$L_2 : 3x - ky - 1 = 0$$

$$L_3 : 5x + 2y - 12 = 0$$

L_1 and L_3 intersect at $(2, 1)$

$\therefore L_1, L_2, L_3$ are concurrent if

$$6 - k - 1 = 0 \text{ or } k = 5$$

For L_1, L_2 to be parallel

$$\frac{1}{3} = \frac{3}{-k} \Rightarrow k = -9$$

For L_2, L_3 to be parallel

$$\frac{3}{5} = \frac{-k}{2} \Rightarrow k = \frac{-6}{5}$$

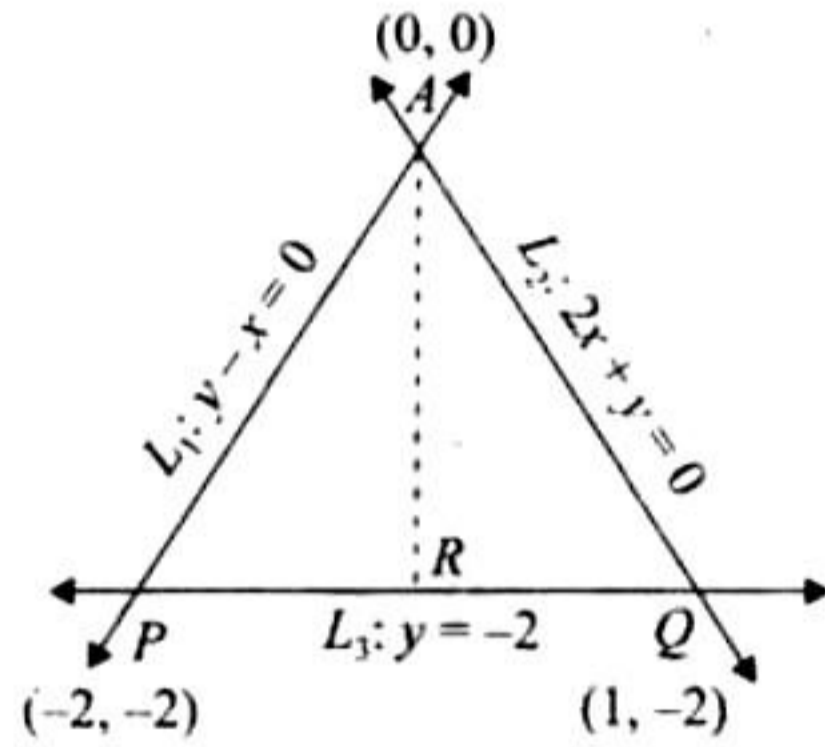
Thus, for $k = 5$, lines are concurrent and for $k = -9, \frac{-6}{5}$, at least two lines are parallel. So, for these values of k , lines will not form triangle.

Obviously, for $k \neq 5, -9, \frac{-6}{5}$, lines form triangle.

Assertion-Reasoning Type

1. c. The point of intersection of L_1 and L_2 is $A(0, 0)$.

Also, $P \equiv (-2, -2), Q \equiv (1, -2)$.

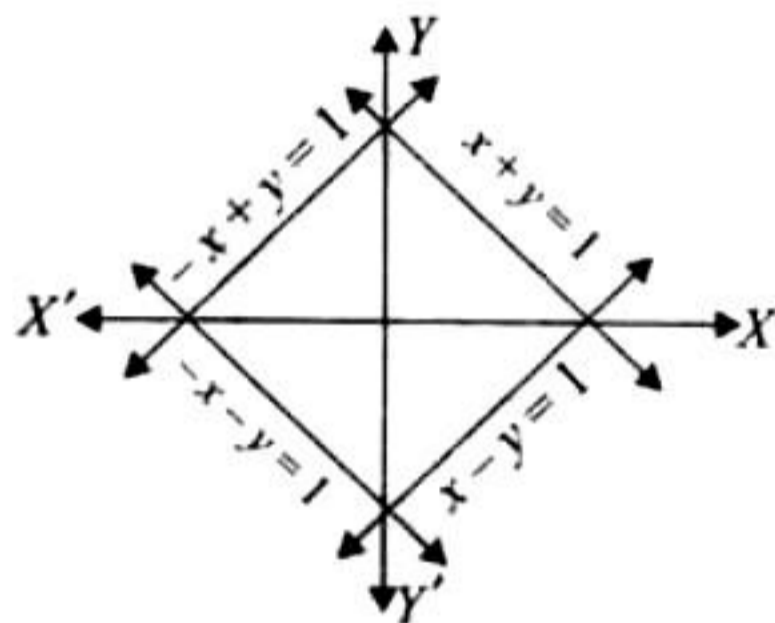


Since AR is the bisector of $\angle PAQ$, R divides PQ in the same ratio as $AP : AQ$. Thus $PR : RQ = AP : AQ = 2\sqrt{2} : \sqrt{5}$. Hence, statement 1 is true. Statement 2 is clearly false.

Fill in the Blanks Type

1. $|x| + |y| = 1$

The curve represents four lines: $x + y = 1, x - y = 1, -x + y = 1, -x - y = 1$.



These lines enclose a square of side equal to the distance between opposite sides $x + y = 1$ and $x + y = -1$, which is given by $(1 + 1)/\sqrt{1 + 1} = \sqrt{2}$. Therefore, the required area is 2 sq. units.

2. Given that $3a + 2b + 4c = 0$.

$$\text{Hence, } \frac{3}{4}a + \frac{1}{2}b + c = 0$$

Hence, the set of lines $ax + by + c = 0$ pass through the point $(3/4, 1/2)$. Therefore, the given lines are concurrent at the point $(3/4, 1/2)$.

3. If a, b, c are in AP, then

$$a + c = 2b$$

$$\text{or } a - 2b + c = 0$$

Hence, $ax + by + c = 0$ passes through $(1, -2)$.

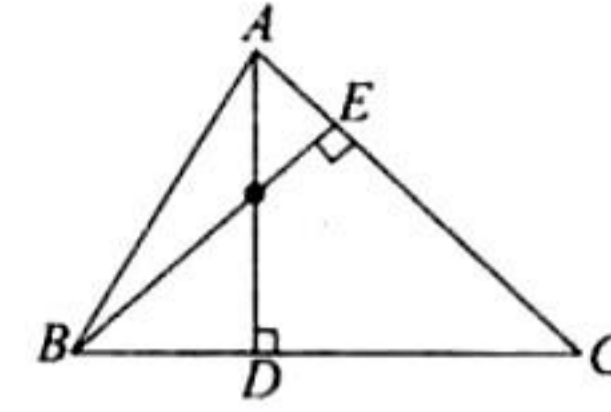
4. The equations of sides of triangle ABC are as follows:

$$AB: x + y = 1$$

$$BC: 2x + 3y = 6$$

$$CA: 4x - y = -4$$

Solving these pairwise, we get the vertices of the triangle as $A(-3/5, 8/5), B(-3, 4), C(-3/7, 16/7)$.



Now, AD is perpendicular to BC and passes through A . Any line perpendicular to BC is $3x - 2y + \lambda = 0$. As it passes through $(-3/5, 8/5)$, we have

$$\frac{-9}{5} - \frac{16}{5} + \lambda = 0$$

$$\text{or } \lambda = 5$$

Hence, the equation of altitude AD is

$$3x - 2y + 5 = 0 \quad \text{(i)}$$

Any line perpendicular to side AC is $x + 4y + \mu = 0$. As it passes through point $B(-3, 4)$, we have

$$-3 + 16 + \mu = 0 \quad \text{or } \mu = -13$$

Therefore, the equation of altitude BE is

$$x + 4y - 13 = 0 \quad \text{(ii)}$$

Now, the orthocenter is the intersection point of (i) and (ii) (AD and BE).

Solving (i) and (ii), we get $x = 3/7, y = 22/7$. Hence, the orthocenter lies in the first quadrant.

5. Let the variable line be

$$ax + by + c = 0 \quad \text{(i)}$$

Then the perpendicular distance of the line from $(2, 0)$ is

$$p_1 = \frac{2a + c}{\sqrt{a^2 + b^2}}$$

The perpendicular distance of the line from $(0, 2)$ is

$$p_2 = \frac{2b + c}{\sqrt{a^2 + b^2}}$$

The perpendicular distance of the line from $(1, 1)$ is

$$p_3 = \frac{a + b + c}{\sqrt{a^2 + b^2}}$$

According to question,

$$p_1 + p_2 + p_3 = 0$$

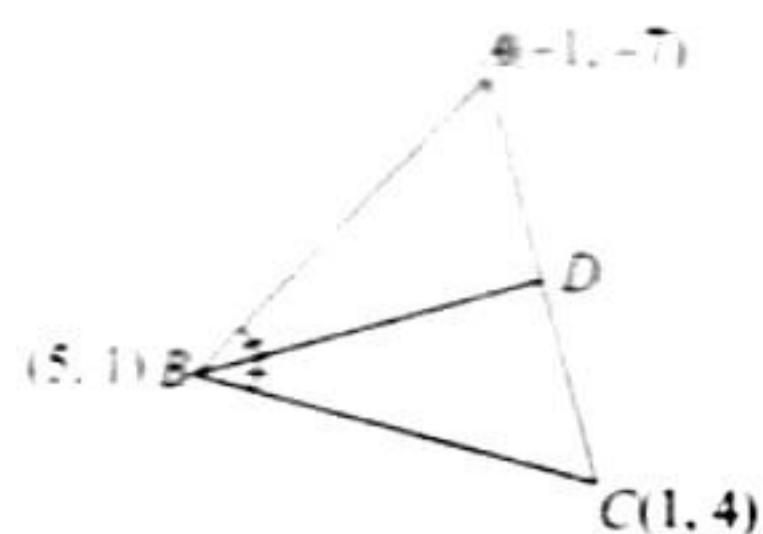
$$\text{or } \frac{2a + c + 2b + c + a + b + c}{\sqrt{a^2 + b^2}} = 0$$

$$\text{or } 3a + 3b + 3c = 0$$

$$\text{or } a + b + c = 0 \quad \text{(ii)}$$

From (i) and (ii), we can say that variable line (i) passes through the fixed point $(1, 1)$.

6. Let BD be the bisector of $\angle ABC$. Then



$$AD : DC = AB : BC$$

and $AB = \sqrt{(5-1)^2 + (-1-7)^2} = 10$

$$BC = \sqrt{(5-1)^2 + (-1-4)^2} = 5$$

$$\therefore AD : DC = 2 : 1$$

Therefore, by section formula, $D = (1/3, 1/3)$. Therefore, the equation of BD is

$$y - 1 = \frac{1/3 - 1}{1/3 - 5}(x - 5)$$

or $y - 1 = \frac{-2/3}{-14/3}(x - 5)$

or $7y - 7 = x - 5$

or $x - 7y + 2 = 0$

True/False Type

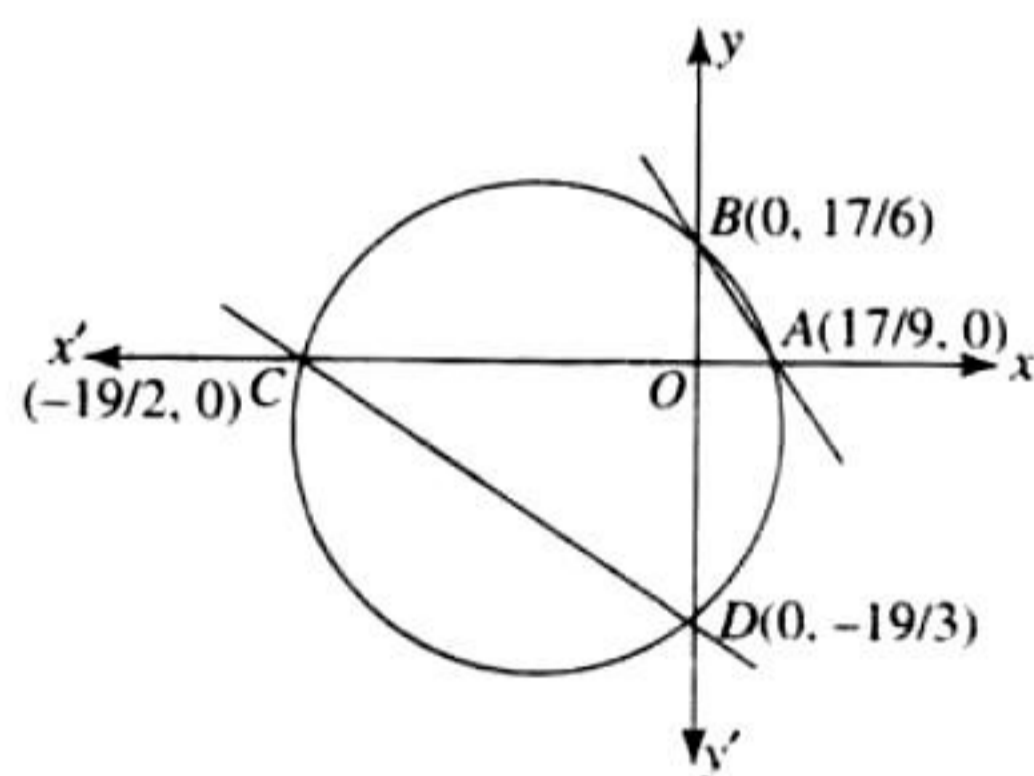
1. For the given lines, we have

$$\begin{vmatrix} 5 & 4 & 0 \\ 1 & 2 & -10 \\ 2 & 1 & 5 \end{vmatrix} = 5(10 + 10) - 4(5 + 20) = 100 - 100 = 0$$

Therefore lines are concurrent

Hence, the statement is true.

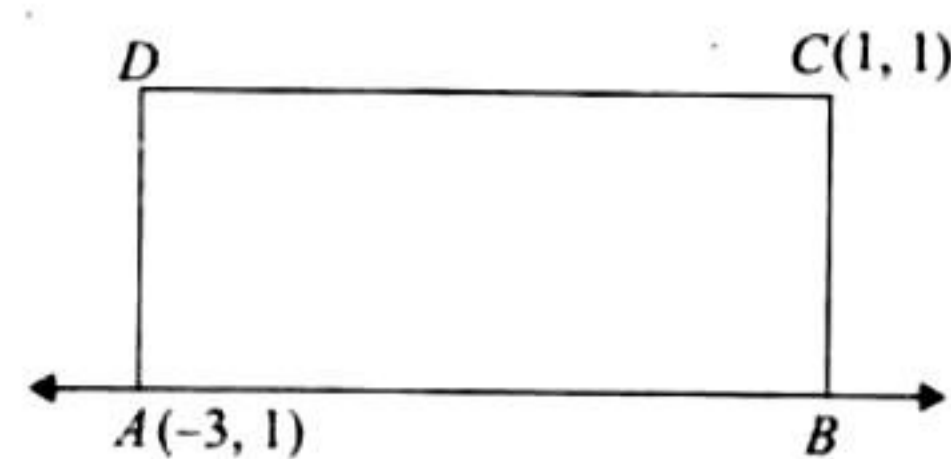
2. Given lines cut the x -axis at $A(17/9, 0)$ and $C(-19/2, 0)$.



The given lines cut the y -axis at $B(0, 17/6)$ and $D(0, -19/3)$. Now, $A, B, C,$ and D are concyclic if $AO \times OC = BO \times OD$, which holds. Hence, the given statement is true.

Subjective Type

1. Let the side AB of rectangle $ABCD$ lies along $4x + 7y + 5 = 0$. As $(-3, 1)$ lies on this line, let it be vertex A . Now, $(1, 1)$ is either vertex C or D .



If $(1, 1)$ is vertex D , then the slope of AD is 0. Hence, AD is not perpendicular to AB . But it is a contradiction as $ABCD$ is a rectangle. Therefore, $(1, 1)$ are the coordinates of vertex C .

CD is a line parallel to AB and passing through C . Therefore, the equation of CD is

$$y - 1 = -\frac{4}{7}(x - 1)$$

or $4x + 7y - 11 = 0$

Also, BC is a line perpendicular to AB and passing through C . Therefore, the equation of BC is

$$y - 1 = \frac{7}{4}(x - 1)$$

or $7x - 4y - 3 = 0$

Similarly, AD is a line perpendicular to AB and passing through $A(-3, 1)$. Therefore, the equation of line AD is

$$y - 1 = \frac{7}{4}(x + 3)$$

or $7x - 4y + 25 = 0$

2. The given lines are

$$x - 2y + 4 = 0 \quad (i)$$

and $4x - 3y + 2 = 0 \quad (ii)$

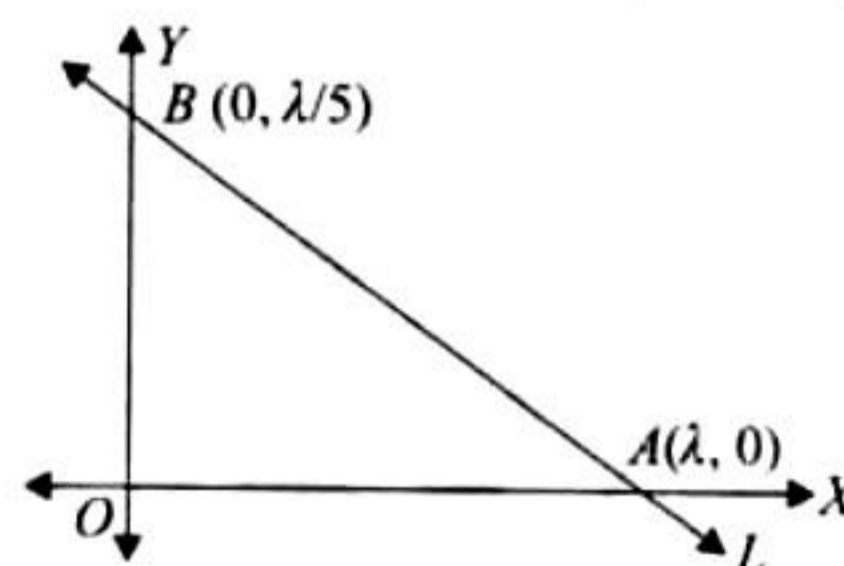
Both the lines have constant terms of same sign. Therefore, the equations of bisectors of the angles between the given lines are

$$\frac{x - 2y + 4}{\sqrt{1 + 4}} = \pm \frac{4x - 3y + 2}{\sqrt{16 + 9}}$$

Here, $a_1 a_2 + b_1 b_2 > 0$. Therefore, taking positive sign on the RHS, we get the obtuse angle bisector as

$$(4 - \sqrt{5})x + (2\sqrt{5} - 3)y - (4\sqrt{5} - 2) = 0 \quad (iii)$$

3. The given line is $5x - y = 1$. Therefore, the equation of line L which is perpendicular to the given line is $x + 5y = \lambda$. This line meets the coordinate axes at $A(\lambda, 0)$ and $B(0, \lambda/5)$.



$$\therefore \text{Area of } \triangle OAB = \frac{1}{2} \times OA \times OB$$

or $5 = \frac{1}{2} \times \lambda \times \frac{\lambda}{5}$

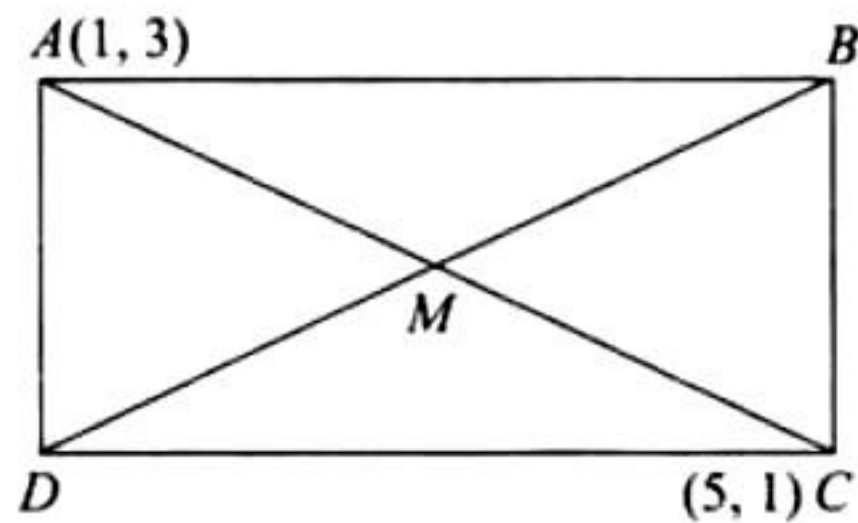
or $\lambda^2 = 5^2 \times 2$

or $\lambda = \pm 5\sqrt{2}$

Hence, the equation of line L is

$$x + 5y - 5\sqrt{2} = 0 \text{ or } x + 5y + 5\sqrt{2} = 0$$

4. Let $ABCD$ be a rectangle and the coordinates of its opposite vertices A and C be $(1, 3)$ and $(5, 1)$, respectively.



Given that other two vertices B and D lie on $y = 2x + c$. Hence, the equation of diagonal BD is $2x - y + c = 0$.

Now, we know that the diagonals of a rectangle bisect each other. Therefore, the midpoint of AC will lie on BD , which is given by

$$M \equiv \left(\frac{1+5}{2}, \frac{3+1}{2} \right) \equiv (3, 2)$$

As $(3, 2)$ lies on $2x - y + c = 0$, we have

$$2 \times 3 - 2 + c = 0$$

or $c = -4$

Hence, the equation of BD is $2x - y - 4 = 0$. Now, the slope of BD is $m = 2$ or $\tan \theta = 2$. Therefore,

$$\cos \theta = \frac{1}{\sqrt{5}}, \sin \theta = \frac{2}{\sqrt{5}}$$

Therefore, the equation of BD in symmetric form is

$$\frac{x-3}{1/\sqrt{5}} = \frac{y-2}{2/\sqrt{5}} = r \quad (i)$$

Now, the length of diagonal AC is $\sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$. As the diagonals of a rectangle are equal in length, $BD = 2\sqrt{5}$.

Also, M is the midpoint of BD . Therefore, $BM = MD = \sqrt{5}$. In order to find B , we use the symmetrical form of line BD with $r = \sqrt{5}$. Therefore,

$$\frac{x-3}{1/\sqrt{5}} = \frac{y-2}{2/\sqrt{5}} = \sqrt{5}$$

or $x = 1 + 3, y = 2 + 2$

or $B \equiv (4, 4)$

Similarly, for point D (being in opposite direction of BD we use $r = -\sqrt{5}$), we get

$$\frac{x-3}{1/\sqrt{5}} = \frac{y-2}{2/\sqrt{5}} = -\sqrt{5}$$

or $x = -1 + 3, y = -2 + 2$

or $x = 2, y = 0$

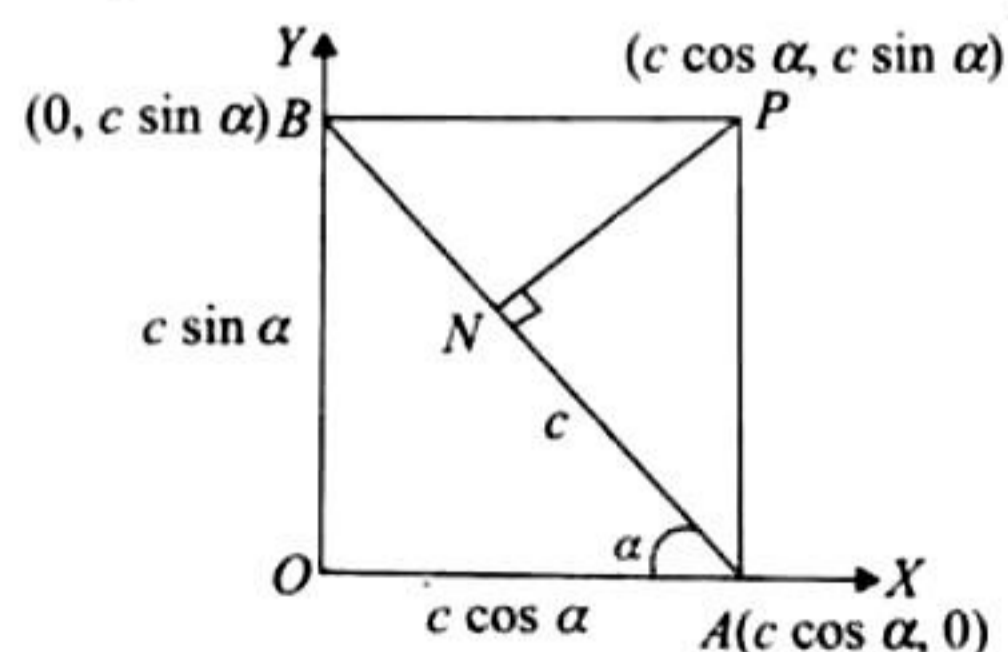
$\therefore D \equiv (2, 0)$

Hence, $c = -4$ and the remaining vertices are $(4, 4)$ and $(2, 0)$.

5. Here, the equation of AB is

$$\frac{x}{c \cos \alpha} + \frac{y}{c \sin \alpha} = 1$$

or $x \sin \alpha + y \cos \alpha = c \sin \alpha \cos \alpha \quad (i)$



The equation of PN (perpendicular to AB and through P) is

$$y - c \sin \alpha = \cot \alpha (x - c \cos \alpha)$$

or $x \cos \alpha - y \sin \alpha = c(\cos^2 \alpha - \sin^2 \alpha) \quad (ii)$

N is the intersection point of (i) and (ii). Solving (i) and (ii), we get

$$x = c \cos^3 \alpha, y = c \sin^3 \alpha$$

$$\therefore \cos \alpha = \left(\frac{x}{c} \right)^{1/3}, \sin \alpha = \left(\frac{y}{c} \right)^{1/3}$$

Therefore, the locus of (x, y) is

$$\left(\frac{x}{c} \right)^{2/3} + \left(\frac{y}{c} \right)^{2/3} = 1$$

$$\text{or } x^{2/3} + y^{2/3} = c^{2/3}$$

6. The given equations of equal sides are

$$7x - y + 3 = 0 \quad (i)$$

$$x + y - 3 = 0 \quad (ii)$$

Let m be the slope of line which passes through the point $(1, -10)$.

Thus, the equation of line will be

$$y + 10 = m(x - 1) \quad (iii)$$

Now, the given lines make same angle with the line in (iii).

Therefore,

$$\left| \frac{m-7}{1+7m} \right| = \left| \frac{m+1}{1-m} \right|$$

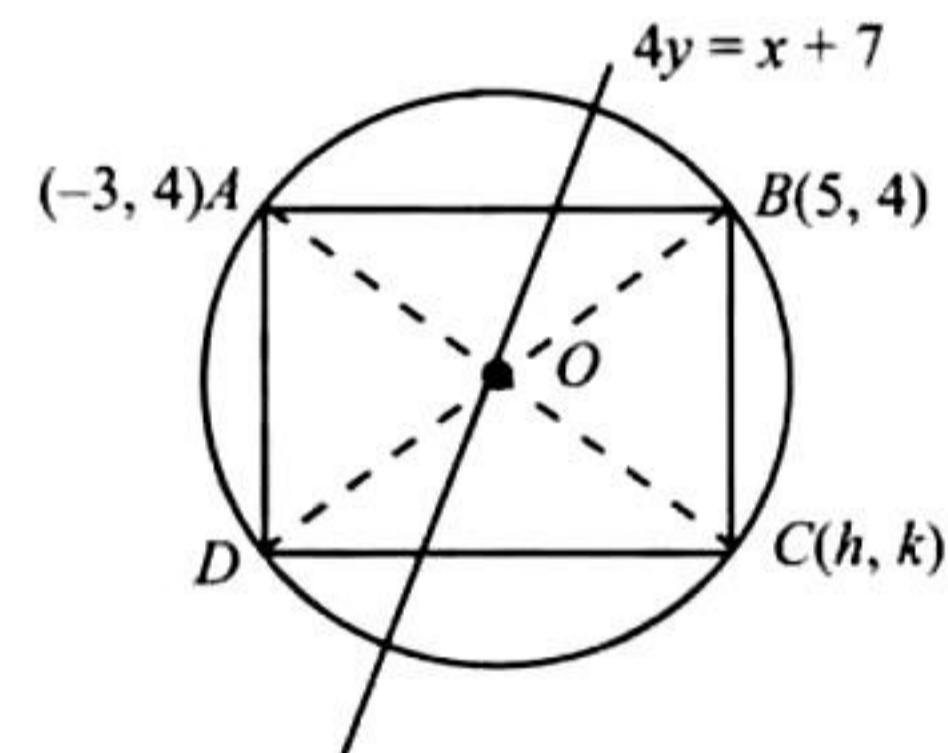
i.e., $m = -3$ or $\frac{1}{3}$

Hence, the required line is

$$y + 10 = -3(x - 1) \text{ or } y + 10 = \frac{1}{3}(x - 1)$$

i.e., $3x + y + 7 = 0$ or $x - 3y - 31 = 0$

7.



From the diagram, $BC \perp AB$.

The slope of AB is 0. Then,

From BC , $h - 5 = 0 \quad (i)$

Also, the midpoint of AC lies on the given diameter. Therefore,

$$4 \left(\frac{k+4}{2} \right) = \frac{h-3}{2} + 7 \quad (ii)$$

From (i) and (ii),

$$h = 5 \text{ and } k = 0$$

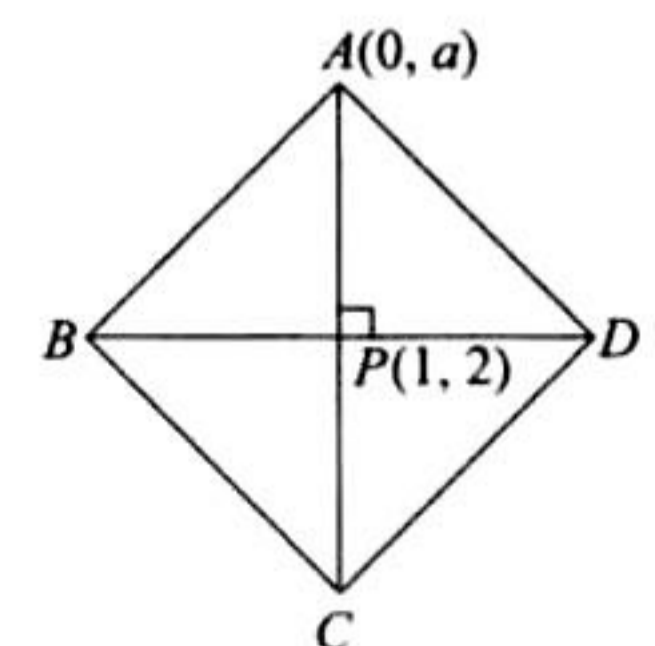
Hence, C is $(5, 0)$. Therefore,

$$BC = 4 \text{ and } AB = 8$$

Therefore, the area of rectangle is $AB \times BC = 32$ sq. units.

8. A being on the y -axis may be chosen as $(0, a)$. The diagonals intersect at $P(1, 2)$.

Again, we know that the diagonals will be parallel to the bisectors of the two given lines $y = x + 2$ and $y = 7x + 3$, i.e.,



$$\frac{x-y+2}{\sqrt{2}} = \pm \frac{7x-y+3}{5\sqrt{2}}$$

or $5x-5y+10 = \pm(7x-y+3)$

or $2x+4y-7=0$ and $12x-6y+13=0$

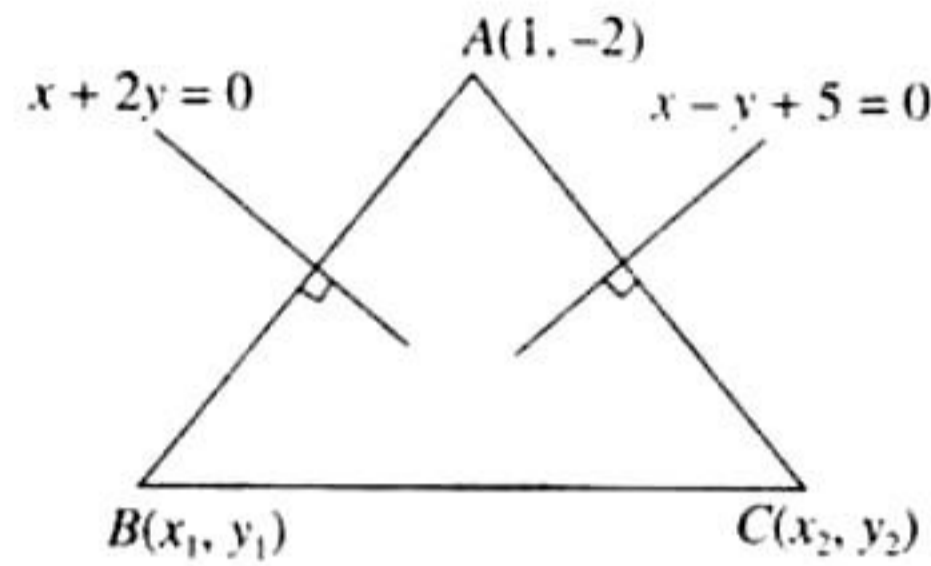
\therefore Slope of diagonals are $m_1 = -1/2$ and $m_2 = 2$. Let diagonal d_1 be parallel to $2x+4y-7=0$ and diagonal d_2 be parallel to $12x-6y+13=0$. The vertex A could be on any of the two diagonals. Hence, the slope of AP is either $-1/2$ or 2 . Therefore,

$$\frac{2-a}{1-0} = 2 \text{ or } -\frac{1}{2}$$

i.e., $a=0$ or $\frac{5}{2}$

Hence, A is $(0, 0)$ or $(0, 5/2)$.

9.



Since the given lines are perpendicular bisectors of the sides as shown in the figure, points B and C are the images of the point A in these lines. Therefore,

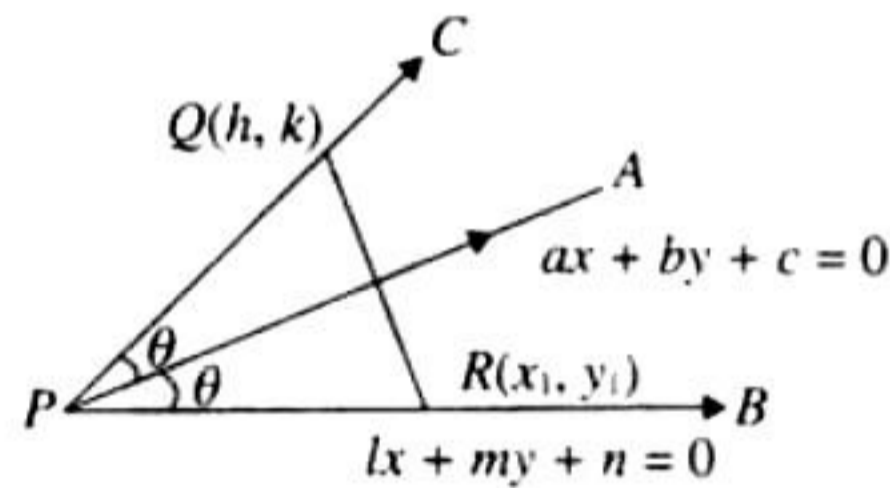
$$\frac{x_1-1}{1} = \frac{y_1+2}{-1} = -\frac{2(1-4)}{1+4}$$

and $\frac{x_2-1}{1} = \frac{y_2+2}{2} = -\frac{2(1+2+5)}{1+1}$

$\therefore B(x_1, y_1) \equiv (11/5, 2/5)$ and $C(x_2, y_2) \equiv (-7, 6)$

Hence, the line passing through the points B and C is $14x+23y-40=0$.

10.



Clearly, AP is the bisector of $\angle CPB$.

Hence, the image of any point Q on line AC in line PA must lie on line PB . Therefore,

$$\frac{x_1-h}{a} = \frac{y_1-k}{b} = -\frac{2(ah+bk+c)}{a^2+b^2}$$

or $x_1 = -\frac{2a(ah+bk+c)}{a^2+b^2} + h$

and $y_1 = -\frac{2b(ah+bk+c)}{a^2+b^2} + k$

Now, $R(x_1, y_1)$ lies on the line $lx+my+n=0$. Therefore,

$$l\left[-\frac{2a(ah+bk+c)}{a^2+b^2} + h\right] + m\left[-\frac{2b(ah+bk+c)}{a^2+b^2} + k\right] + n = 0$$

Hence, the locus of Q is

$$2(al+mb)(ax+by+c) - (a^2+b^2)(lm+my+n) = 0$$

11. Let BC be taken as the x -axis with the origin at D . Let A be taken on y -axis. Let $BC = 2a$. Now $AB = AC$, then the coordinates of B and C are $(-a, 0)$ and $(a, 0)$, respectively. Let $DA = h$. Then the coordinates of A are $(0, h)$.

Hence, the equation of AC is

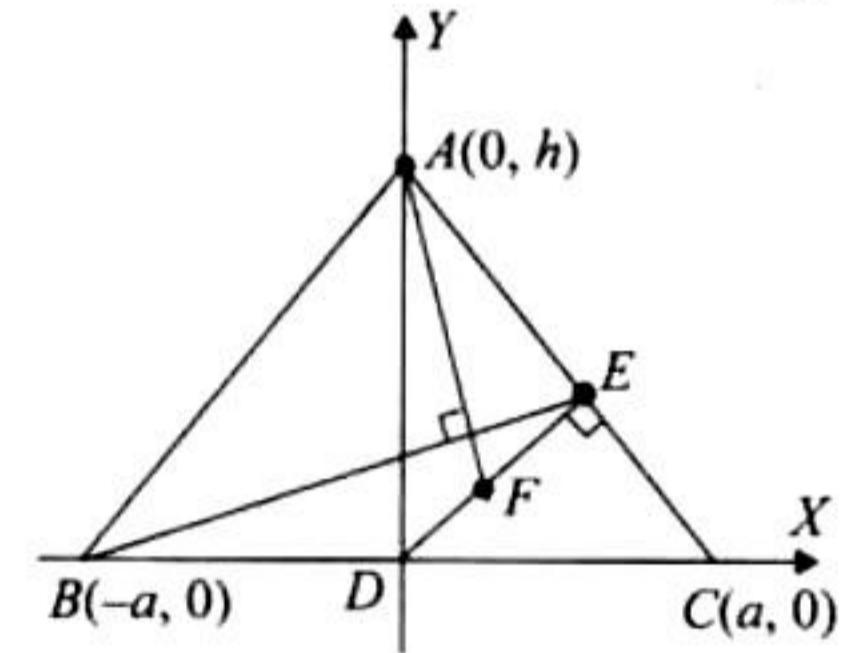
$$\frac{x}{a} + \frac{y}{h} = 1 \tag{i}$$

and the equation of DE perpendicular to AC and passing through the origin is

$$\frac{x}{h} - \frac{y}{a} = 0$$

or $x = \frac{hy}{a}$

(ii)



Solving (i) and (ii), we get the coordinates of E as follows:

$$\frac{hy}{a^2} + \frac{y}{h} = 1$$

or $h^2y + a^2y = a^2h$

or $y = \frac{a^2h}{a^2+h^2}$

$\therefore x = \frac{ah^2}{a^2+h^2}$

$\therefore E \equiv \left(\frac{ah^2}{a^2+h^2}, \frac{a^2h}{a^2+h^2}\right)$

Since F is the midpoint of DE , its coordinates are

$$\left(\frac{ah^2}{2(a^2+h^2)}, \frac{a^2h}{2(a^2+h^2)}\right)$$

The slope of AF is

$$m_1 = \frac{h - \frac{a^2h}{2(a^2+h^2)}}{0 - \frac{ah^2}{2(a^2+h^2)}}$$

$$= \frac{2h(a^2+h^2) - a^2h}{-ah^2}$$

$$= -\frac{a^2+2h^2}{ah} \tag{i}$$

and the slope of BE is

$$m_2 = \frac{\frac{a^2h}{a^2+h^2} - 0}{\frac{ah^2}{a^2+h^2} + a}$$

$$= \frac{a^2h}{ah^2 + a^3 + ah^2}$$

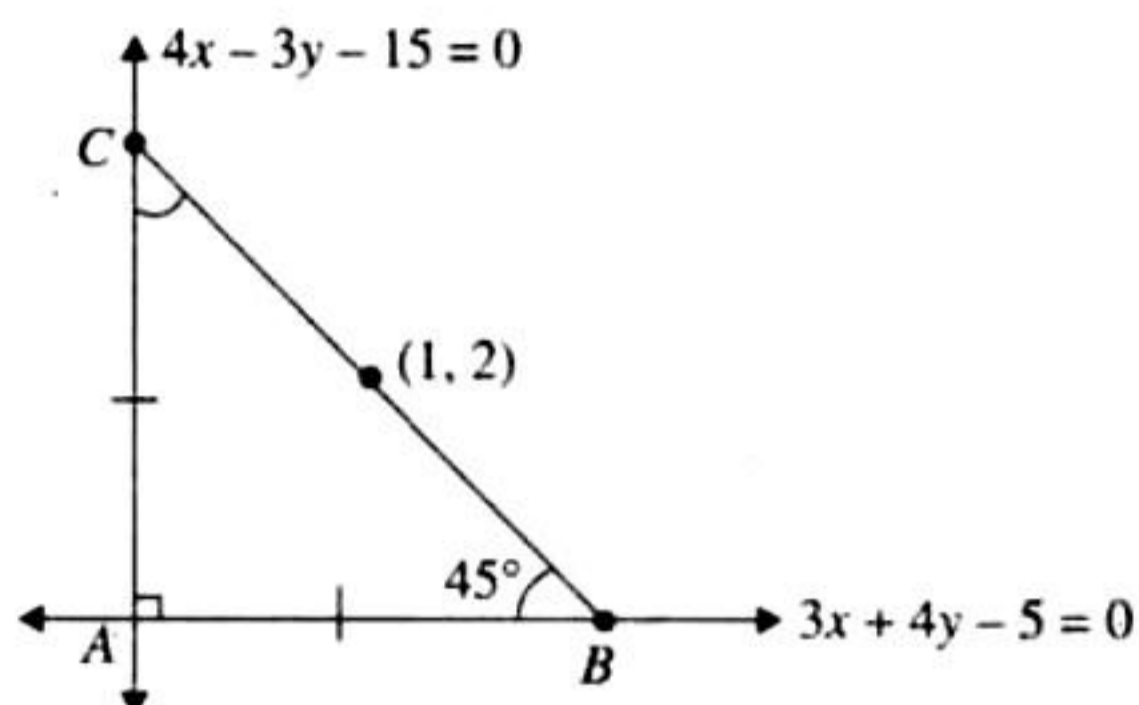
$$= \frac{ah}{a^2+2h^2} \tag{ii}$$

From (i) and (ii), we have

$$m_1 m_2 = -1$$

or $AF \perp BE$

12. The given straight lines are $3x + 4y = 5$ and $4x - 3y = 15$. Clearly, these straight lines are perpendicular to each other ($m_1 m_2 = -1$), and intersect at A . Now, B and C are the points on these lines such that $AB = AC$ and BC passes through $(1, 2)$. From the figure, it is clear that $\angle B = \angle C = 45^\circ$.



Let the slope of BC be m . Then,

$$\tan 45^\circ = \left| \frac{m + 3/4}{1 - (3/4)m} \right|$$

or $4m + 3 = \pm(4 - 3m)$

i.e., $4m + 3 = 4 - 3m$ or $4m + 3 = -4 + 3m$

i.e., $m = \frac{1}{7}$ or $m = -7$

Hence, the equation of BC is

$$y - 2 = \frac{1}{7}(x - 1) \text{ or } y - 2 = -7(x - 1)$$

i.e., $7y - 14 = x - 1$ or $y - 2 = -7x + 7$

i.e., $x - 7y + 13 = 0$ or $7x + y - 9 = 0$

13. Given lines are

$$2x + y - 3 = 0 \quad (i)$$

$$\text{and } 2x + y - 5 = 0 \quad (ii)$$

Given $AB = 2$

$AL =$ distance between parallel

$$\text{lines (i) and (ii)} = \frac{|-3 + 5|}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}}$$

$$\text{From } \triangle ALB, \sin \alpha = \frac{AL}{AB} = \frac{\frac{2}{\sqrt{5}}}{2} = \frac{1}{\sqrt{5}}$$

$$\therefore \tan \alpha = \frac{1}{2}$$

Now slope of given lines is -2 .

Let the slope of AB be m . Then

$$\tan \alpha = \frac{|m - (-2)|}{|1 + m(-2)|}$$

$$\Rightarrow \pm \frac{1}{2} = \frac{m + 2}{1 - 2m}$$

$$\Rightarrow 1 - 2m = 2m + 4 \text{ or } 2m - 1 = 2m + 4$$

$$\Rightarrow m = -3/4 \text{ or } m = \infty$$

Therefore, equation of lines can be $x - 2 = 0$ or $(y - 3) = (-3/4)$

$(x - 2)$

$$\text{or } 3x + 4y - 18 = 0$$

14. The given curve is

$$3x^2 - y^2 - 2x + 4y = 0 \quad (i)$$

Let $y = mx + c$ be the chord of curve (i) which subtends an angle of 90° at the origin. Then the combined equations of lines joining the points of intersection of curve (i) and chord $y = mx + c$ to the origin can be obtained by making the equation of curve homogeneous with the help of the equation of chord as

$$3x^2 - y^2 - 2x\left(\frac{y - mx}{c}\right) + 4y\left(\frac{y - mx}{c}\right) = 0$$

$$\text{or } 3cx^2 - cy^2 - 2xy + 2mx^2 + 4y^2 - 4mxy = 0$$

$$\text{or } (3c + 2m)x^2 - 2(1 + 2m)xy + (4 - c)y^2 = 0$$

As the lines represented by this pair are perpendicular to each other, we must have

$$\text{Coefficient of } x^2 + \text{Coefficient of } y^2 = 0$$

$$\text{Hence, } 3c + 2m + 4 - c = 0$$

$$\text{or } -2 = m + c$$

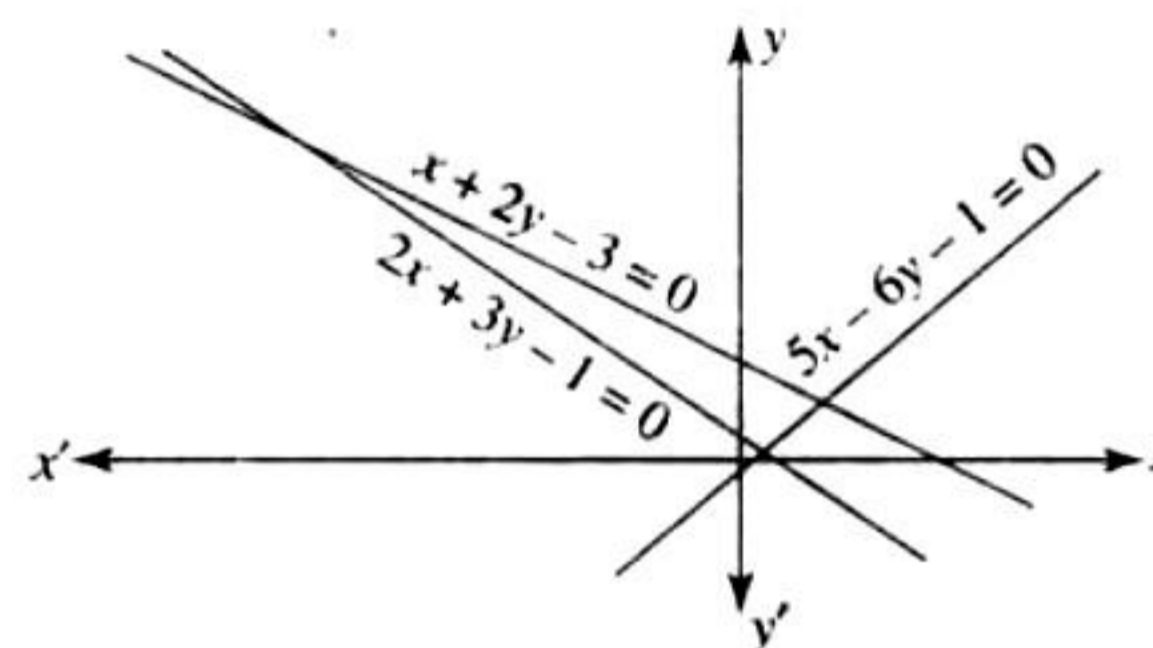
Comparing this result with $y = mx + c$, we can see that $y = mx + c$ passes through $(1, -2)$.

15. Given lines are $L_1 : 2x + 3y - 1 = 0$

$$L_2 : 5x - 6y - 1 = 0$$

$$L_3 : x + 2y - 3 = 0$$

These lines can be drawn on coordinate axes as shown:



Clearly, from the figure, origin O and point $P(\alpha, \alpha^2)$ must lie on the opposite sides w.r.t $2x + 3y - 1 = 0$.

$$L_1(0, 0) : -1 < 0$$

$$\Rightarrow L_1(\alpha, \alpha^2) : 2\alpha + 3\alpha^2 - 1 > 0$$

$$\Rightarrow (3\alpha - 1)(\alpha + 1) > 0$$

$$\Rightarrow \alpha < -1 \text{ or } \alpha > 1/3 \quad (i)$$

O and the point (α, α^2) must lie to the same side w.r.t. $x + 2y - 3 = 0$.

$$L_3(0, 0) : -3 < 0$$

$$\Rightarrow L_3(\alpha, \alpha^2) : \alpha + 2\alpha^2 - 3 < 0$$

$$\Rightarrow (2\alpha + 3)(\alpha - 1) < 0$$

$$\Rightarrow -3/2 < \alpha < 1 \quad (ii)$$

Again O and $P(\alpha, \alpha^2)$ must lie to the same side w.r.t. $5x - 6y - 1 = 0$

$$\Rightarrow L_2(0, 0) : -1 < 0$$

$$\Rightarrow L_2(\alpha, \alpha^2) : 5\alpha - 6\alpha^2 - 1 < 0$$

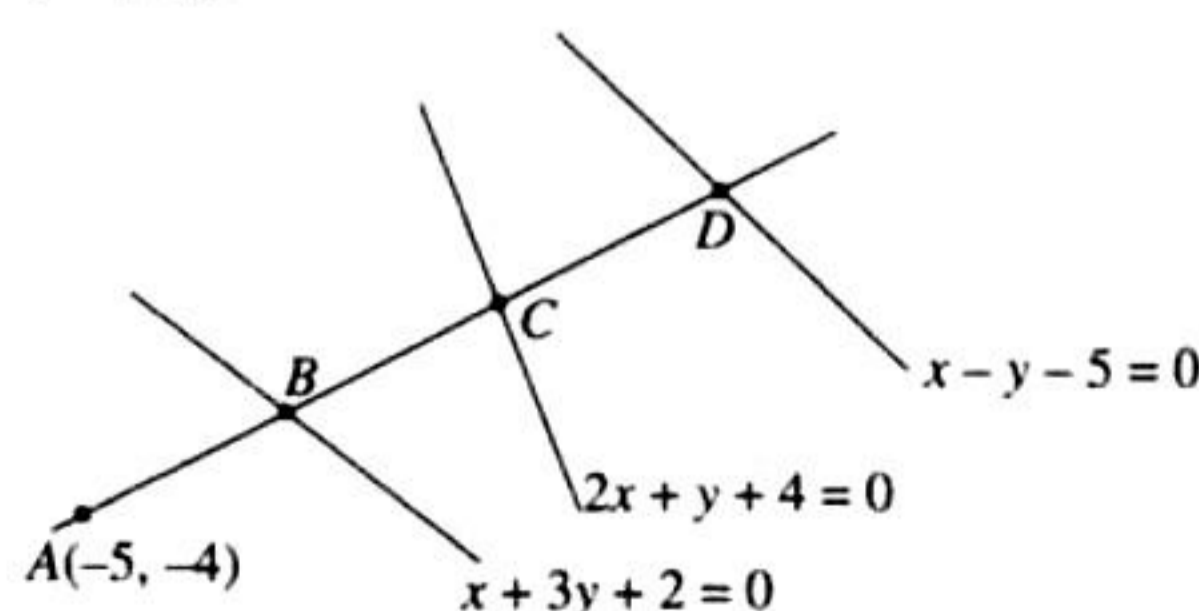
$$\Rightarrow 6\alpha^2 - 5\alpha + 1 > 0$$

$$\Rightarrow (3\alpha - 1)(2\alpha - 1) > 0$$

$$\Rightarrow \alpha < 1/3 \text{ or } \alpha > 1/2 \quad (iii)$$

From (i), (ii) and (iii) common values of α are $(-3/2, -1) \cup (1/2, 1)$.

- 16.



Let θ be the inclination of line through $A(-5, -4)$.

Therefore, equation of this line is $\frac{x+5}{\cos \theta} = \frac{y+4}{\sin \theta} = r$ (i)

Any point on this line at distance r from point $A(-5, -4)$ is given by

$$(-5 + r \cos \theta, -4 + r \sin \theta)$$

If $AB = r_1$, $AC = r_2$ and $AD = r_3$, then

$$B(r_1 \cos \theta - 5, r_1 \sin \theta - 4)$$

$$C(r_2 \cos \theta - 5, r_2 \sin \theta - 4)$$

$$D(r_3 \cos \theta - 5, r_3 \sin \theta - 4)$$

But B lies on $x + 3y + 2 = 0$. Therefore,

$$r_1 \cos \theta - 5 + 3r_1 \sin \theta - 12 + 2 = 0$$

or $\frac{15}{\cos \theta + 3 \sin \theta} = r_1$

or $\frac{15}{AB} = \cos \theta + 3 \sin \theta$ (i)

As C lies on $2x + y + 4 = 0$, we have

$$2(r_2 \cos \theta - 5) + (r_2 \sin \theta - 4) + 4 = 0$$

or $r_2 = \frac{10}{2 \cos \theta + \sin \theta} = AC$

or $\frac{10}{AC} = 2 \cos \theta + \sin \theta$ (ii)

Similarly, D lies on $x - y - 5 = 0$. Therefore,

$$r_3 \cos \theta - 5 - r_3 \sin \theta + 4 - 5 = 0$$

or $r_3 = \frac{6}{\cos \theta - \sin \theta} = AD$

or $\frac{6}{AD} = \cos \theta - \sin \theta$ (iii)

Now, given that

$$\left(\frac{15}{AB}\right)^2 + \left(\frac{10}{AC}\right)^2 = \left(\frac{6}{AD}\right)^2$$

or $(\cos \theta + 3 \sin \theta)^2 + (2 \cos \theta + \sin \theta)^2 = (\cos \theta - \sin \theta)^2$
[Using (i), (ii), and (iii)]

or $4 \cos^2 \theta + 9 \sin^2 \theta + 12 \sin \theta \cos \theta = 0$

or $2 \cos \theta + 3 \sin \theta = 0$

or $\tan \theta = -\frac{2}{3}$

Hence, the equation of the required line is

$$y + 4 = -\frac{2}{3}(x + 5)$$

or $3y + 12 = -2x - 10$

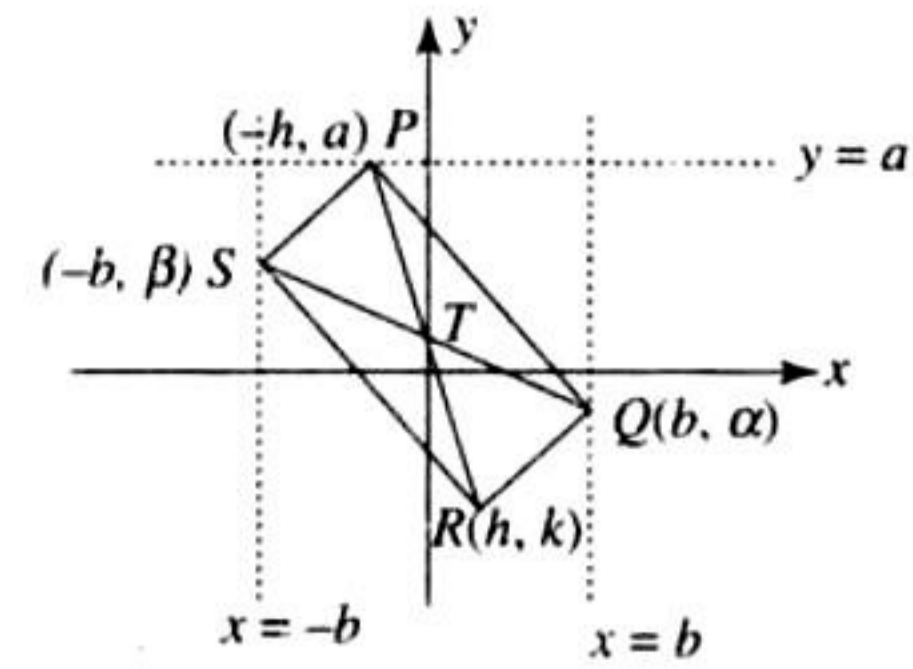
or $2x + 3y + 22 = 0$

17. Let the coordinate of Q be (b, α) and that of S be $(-b, \beta)$. Let PR and SQ intersect each other at T .

$\therefore T$ is the mid point of SQ

Since diagonals of a rectangle bisect each other, x co-ordinates of P is $-h$.

As P lies on $y = a$, therefore coordinates of P are $(-h, a)$.



Given that PQ is parallel to $y = mx$ and slope of $PQ = m$.

$$\therefore \frac{\alpha - a}{b + h} = m$$

$$\Rightarrow \alpha = a + m(b + h) \quad (1)$$

Also, $RQ \perp PQ \Rightarrow$ slope of $RQ = -\frac{1}{m}$

$$\therefore \frac{k - \alpha}{h - b} = -\frac{1}{m} \Rightarrow \alpha = k + \frac{1}{m}(h - b) \quad (2)$$

From (1) and (2), we get

$$a + m(b + h) = k + \frac{1}{m}(h - b)$$

$$\Rightarrow am + m^2(b + h) = km + (h - b)$$

$$\Rightarrow (m^2 - 1)h - mk + b(m^2 + 1) + am = 0$$

Therefore, locus of (h, k) is

$$(m^2 - 1)x - my + b(m^2 + 1) + am = 0$$

18. The line $y = mx$ meets the given lines at

$$P\left(\frac{1}{m+1}, \frac{m}{m+1}\right) \text{ and } Q\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$$

Hence, the equation of L_1 is

$$\left(y - \frac{m}{m+1}\right) = 2\left(x - \frac{1}{m+1}\right)$$

or $y - 2x - 1 = -\frac{3}{m+1} \quad (i)$

and that of L_2 is

$$\left(y - \frac{3m}{m+1}\right) = -3\left(x - \frac{3}{m+1}\right)$$

or $y + 3x - 3 = \frac{6}{m+1} \quad (ii)$

From (i) and (ii), eliminating m , we get

$$\frac{y - 2x - 1}{y + 3x - 3} = -\frac{1}{2}$$

or $x - 3y + 5 = 0$

which is a straight line.

19. Let the equation of the line be

$$(y - 2) = m(x - 8), \text{ where } m < 0$$

$$\therefore P \equiv \left(8 - \frac{2}{m}, 0\right) \text{ and } Q \equiv (0, 2 - 8m)$$

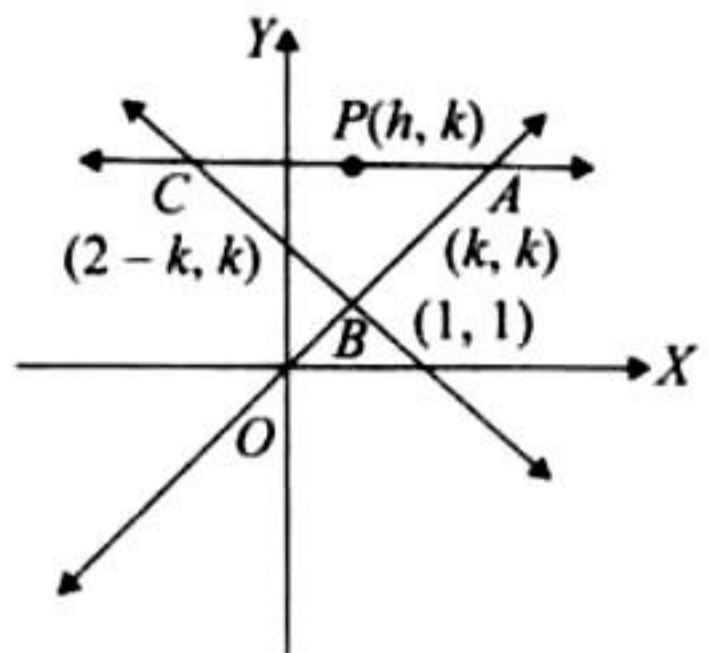
Now, $OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$

$$= 10 + \frac{2}{-m} + (-8m)$$

$$\geq 10 + 2\sqrt{\frac{2}{-m} \times (-8m)} \geq 18$$

(using AM \geq GM)

20. A line passing through $P(h, k)$ and parallel to the x -axis is
 $y = k$ (i)
 The other two lines given are
 $y = x$ (ii)
 and $x + y = 2$ (iii)
 Let ABC be the triangle formed by the points of intersection of lines (i), (ii), and (iii), as shown in the figure.



Then, $A \equiv (k, k)$, $B \equiv (1, 1)$, and $C \equiv (2 - k, k)$. Therefore,

$$\text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} k & k & 1 \\ 1 & 1 & 1 \\ 2 - k & k & 1 \end{vmatrix} = 4h^2$$

Operating $C_1 \rightarrow C_1 - C_2$, we get

$$\frac{1}{2} \begin{vmatrix} 0 & k & 1 \\ 0 & 1 & 1 \\ 2 - 2k & k & 1 \end{vmatrix} = 4h^2$$

$$\therefore \frac{1}{2} (2 - 2k)(k - 1) = 4h^2$$

$$\therefore (k - 1)^2 = 4h^2$$

$$\therefore k - 1 = 2h \text{ or } k - 1 = -2h$$

$$\therefore k = 2h + 1 \text{ or } k = -2h + 1$$

Hence, the locus of (h, k) is

$$y = 2x + 1 \text{ or } y = -2x + 1$$